A General SCM Optimization Model and Software

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Abstract: - Making optimal decisions made about the supply chain is a key task of enterprise management. Capacity allocation, distribution, transportation are to be optimized. A mixed integer programming model is developed, and briefly shown here, together with a decision supporting software that helps the management in decision making. Complex cost function including stepwise functions, either/or type minimum constraints, and even homogeneous transport constraints are involved. Large scale problem cases related to European scale international enterprizes are solved.

Key-Words: - Supply chain, SCM, Optimization, MIP, AIMMS, forecasting, collaborative planning

1 Introduction

Controlling and managing of a production enterprise can be decomposed to levels as shown in Fig.1. Supply Chain Management (SCM) is the top level in this hierarchy. Quality of SCM design constraints all the lower level designs and, in this way, their profitability.

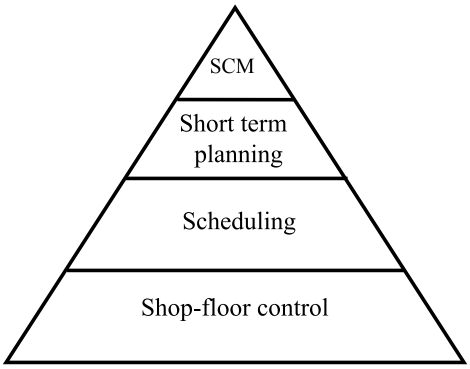

Fig.1. Desision hierarchy

According to [1], SCM includes decisions on

- number, location, capacity and type of the production plants, suppliers, and distribution storages;
- amount of each of the applied row materials and produced products,
- transportation routes;
- amount of each of the row materials and products transported between suppliers, plants, stores, and customers;
- amount of each of the stored row materials, intermediate and final products.

Distribution and allocation decisions are involved in SCM, and no generally accepted standard model is known. Here we present an efficient and easy-to-use model and software tool. This tool has originally been developed to help managing a European supply chain network of a beer production company (SAB Miller) to which Hungarian companies, like Dreher, also belong to, further developed for the special needs of another European network producing pet food products (Provimi Pet Food), and is applicable in a wide area of industrial sectors.

2 SCM Optimization Model

2.1 Supply chain structure

Two main decision levels, the level of plants and the level of customers, are considered in the supply chain model. Customers have certain demands for products which can be produced in the plants. The production process is modeled as being consisted of two consecutive steps. Semi-products are produced in production lines. These semi-products are subsequently modified, mixed, and packaged in so-called packaging lines. This packaged product is the final product of the plants, and is transported to the customers. The semi-products can be transported between plants, i.e. a packaging line can package semi-products produced in another plant.

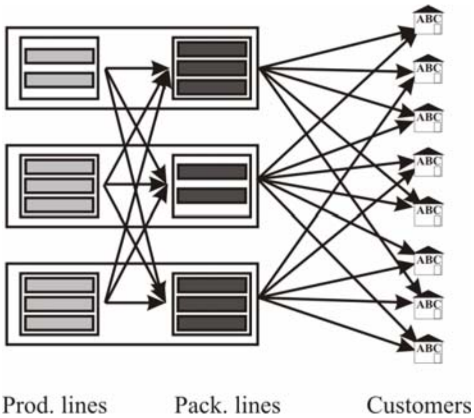

Fig.2. Supply chain model

The level of suppliers is omitted from the model. The level of warehouses is considered merely in the transportation costs from plants to customers. If a product has to be stored in a warehouse during the transportation from a certain plant to a certain customer then the specific transportation cost of that route is increased with the cost of storage.

Therefore, three levels are considered in the supply chain model: (1) production lines of plants, (2) packaging lines of plants, and (3) customers.

2.2 Model characteristics

The target is to minimize the sum of transportation costs, production costs, and packaging costs. The transportation costs are proportional to the amount of transported semi-products and products. Production and packaging costs consist of variable cost proportional to the amount of the products, and fix costs. Fix costs are given by stepwise constant functions, as is shown in Fig.3. Intervals over which the fix costs are defined can be specified according to the working shifts, for example.

Beside maximum constraints on capacity, 'either/or' minimum constraints are considered at the production lines level, at the packaging lines level, at the semi-product transport, and at the product transport. In each case, existence of a minimum constraint means that the quantity of produced /packaged / transported semi-product / product either has to reach a defined minimum level or has to be zero. In each case this minimum constraint expresses the economical consideration that it is not worth to produce, package, or transport, below a minimum quantity (e.g. one truck, in the case of transportation).

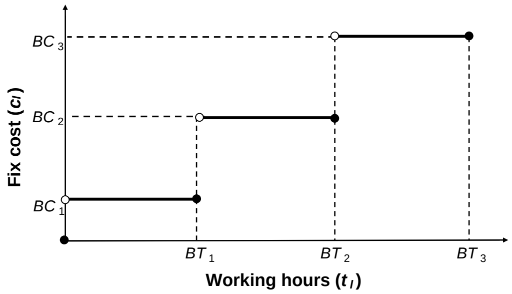

Fig.3. Stepwise cost function

A special feature is the possibility of applying homogenous transport constraints. This means that any amount of a given product is transported to a customer from a single, not *a priori* assigned, plant even if that product can be produced in several plants. Homogenous transport is specified in order to prevent the customer from receiving a mixture of different quality products that might occur because different raw materials and processing methods are applied at the different plants.

2.3 Mathematical model

The supply chain problem is first formulated as a Generalized Disjunctive Programming (GDP) model using continuous and logical variables, then it is transformed into a Mixed Integer Linear Programming (MILP) model using binary variables instead of the logical ones.

The mathematical models incorporate mass balance equations, capacity constraints, the cost functions, and the special constraints mentioned in the previous section. The mass balance equations, the capacity constraints, and the variable cost functions, are all linear and need not be explained here. How the special constraints are formulated is explained below.

2.3.1 Complex cost functions

The fix cost c_l of production or packaging line l depends on its working hours t_l. The range of variable t_l is subdivided to a number of shifts s. The time moment borders of these shifts are given by the upper bound BT_s of shift s,. The constant fix cost of the line in shift s is given by parameter BC_s.

In the GDP representation, logical variables $zs_{l,s}$ are assigned to the shifts. If the value of the independent variable is in or above shift s then $zs_{l,s}$=1, else $zs_{l,s}$=0:

$$\begin{bmatrix} zs_{l,0} \\ t_l = 0 \\ c_l = 0 \end{bmatrix} \underset{s\geq 1}{\oplus} \begin{bmatrix} zs_{l,s} \\ BT_{l,s-1} < t_l \leq BT_{l,s} \\ c_l = BC_{l,s} \end{bmatrix} \quad \forall l, s \qquad (1)$$

(Character ⊕ denotes the logical operation 'exclusive or' or XOR.) This logical equation is transformed into algebraic one using the method of [2]:

$$t_l \leq \sum_s \left(ys_{l,s} \cdot DT_{l,s} \right) \qquad \forall l \qquad (2)$$

$$c_l \geq \sum_s \left(ys_{l,s} \cdot DC_{l,s} \right) \qquad \forall l \qquad (3)$$

$$ys_{l,s} \leq ys_{l,s-1} \qquad \forall l, s \geq 2 \qquad (4)$$

where $DT_{l,s}$ is the length of shift s at line l, $DC_{l,s}$ is the fix cost increment of shift s of line l related to the fix cost of the previos shift $s-1$, and $ys_{l,s}$ is a binary variable.

2.3.2 'Either/or' minimum constraints

How these constraints are formulated is exemplified with the minimum product transport constraint. The $qt_{p,c,pr}$ transported quantity of product pr from plant p to customer c either has to be equal to zero or not smaller than a minimum transport quantity $QTMIN_{p,c,pr}$. In the GDP formulation of this constraint, logical variable $zt_{p,c,pr} \in$ {true, false} denotes whether there is product transport on the given route:

$$\begin{bmatrix} \neg zt_{p,c,pr} \\ qt_{p,c,pr} = 0 \end{bmatrix} \vee \begin{bmatrix} zt_{p,c,pr} \\ qt_{p,c,pr} \geq QTMIN_{p,c,pr} \end{bmatrix} \quad \forall p, c, pr \qquad (5)$$

This logical constraint is transformed into algebraic form using binary variables $yt_{p,c,pr} \in \{0, 1\}$ instead of the logical $zt_{p,c,pr}$ ones:

$$qt_{p,c,pr} \geq QTMIN_{p,c,pr} \cdot yt_{p,c,pr} \qquad \forall p, c, pr \qquad (6)$$

2.3.3 Homogeneous transport constraints

The homogenous transport of product pr to customer c can be defined as a logical constraint using logical variable $zh_{p,c,pr}$. If $zh_{p,c,pr}$ is true, i.e. if product pr is transported only from plant p to customer c, then the transported quantity $qt_{p,c,pr}$ is equal to the demand $D_{c,pr}$ of customer c for product pr, and the transported quantity from all the other $p2 \neq p$ plants is zero.

$$\begin{bmatrix} zh_{p,c,pr} \\ qt_{p,c,pr} = D_{c,pr} \end{bmatrix} \rightarrow \begin{bmatrix} \neg zh_{p2\neq p,c,pr} \\ qt_{p2\neq p,c,pr} = 0 \end{bmatrix} \quad \forall p, c, pr \qquad (7)$$

This logical constraint is transformed into algebraic form using the Convex Hull technique:

$$qt_{p,c,pr} = D_{c,pr} \cdot yh_{p,c,pr} \qquad \forall p, c, pr \qquad (8)$$

$$\sum_p yh_{p,c,pr} = 1 \qquad \forall p, c, pr \qquad (9)$$

where $yh_{p,c,pr}$ is a binary variable.

2.4 Optimization method

The solution of the problem is implemented in AIMMS modelling language [3] using CPLEX as MILP solver. AIMMS has been chosen because of its advanced graphical user interface, and because the most modern solvers can be attached, and the program can be easily extended and developed in case of changing requirements.

In order to facilitate the optimization, preliminary procedures running before the optimization of the main model are developed. First, the tightest bounds of the variables are computed using a Linear Programming (LP) model. Then a feasibility check is performed by solving a simplified LP problem generated from the main MILP model. If the problem is infeasible then the program provides the user with information about the possible reasons of infeasibility. If the problem is feasible then initial values are calculated by solving the relaxed LP model of the main MILP model. As a result of this initialization, the solution procedure of the main MILP model starts from a near optimal point, and thus the solution time is decreased. Finaly, the main MILP model is solved.

2.5 Computation results

The supply chain problems of SAB Miller and Provimi Pet Food involve several plants, with producing and packaging lines, in different countries. For example, the pet food problem containts at least eight plants containing one to three production and packaging lines, more than 80 customers, and more than 800 products.

Solving the main MILP model takes no more than 15 seconds on a PC (2.6 MHz CPU, 512 Mb RAM), and solving the whole problem (including the generation of each mathematical model) takes no more than 5 minutes with relative optimality tolerance (gap) 10^{-13}.

The developed MILP model is tested on three example data sets of different sizes. Main characteristics of the example data are shown in Table 1, where Sz: Size of the model, S/M/L: Small, Middle, Large; Pd/Pc/Cr: number of Production lines, Packaging lines, and Customers, respectively; Eqs: number of Qquations,; Vars: number of Variables; Bins: number of Binary variables.

The effect of including the special constraints (complex cost function equations, 'either/or' minimum constraints, and homogeneous transport constraints) are tested with different versions of the MILP model. Model 1 (M1) contains the material balances, the capacity constraints, and the cost functions only. Model 2 (M2) includes all the constraints of M1, and the minimum constraints as well. Model 3 (M3) includes all the constraints of M1, and the homogenous transport constraints as well. Model 4 (M4) contains all the above mentioned constraints. The solution time data (in CPU sec) at relative optimality tolerance (gap) 10^{-13} are collected in Table 2 and visualized in Fig.4.

Table 1. Characteristics of the example data sets

Sz	Pd/Pc/Cr	Prd	Eqs	Vars	Bins
S	3/5/19	201	4928	4659	1800
M	6/11/45	299	17365	19728	13608
L	13/21/83	856	88240	99707	79826

Table 2. Computation time of model versions

Sz	M1	M2	M3	M4
S	0.02	0.05	0.02	0.07
M	0.08	0.20	0.11	0.23
L	1.06	4.58	1.23	6.47

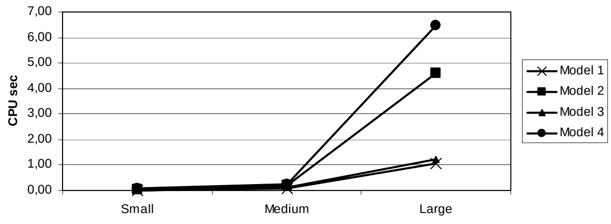

Fig.4. Computation time

Extending Model 1 with the minimum constraints (Model 2) increases the solution time at least with 150%. But including merely the homogenous transport constraints extension (Model 3) increases the solution time just slightly. Homogenous transport constraints are stricter constraints than minimum contraints but they probably cause smaller increase in solution time because (1) this extension means less number of constraints containing binary variables, and (2) their proper Convex Hull formulation results in smaller search space during the optimization.

2.7 Development and installation

AIMMS has been chosen as a development environment by Optasoft because of its advantageous characteristics, namely:

- AIMMS facilitates both a modelling environment and a programming language with usual support tools like syntax check, debugging, etc.
- Independent MP solvers can be attached to AIMMS, and the basic configuration includes useful LP, MILP, and NLP solvers.
- AIMMS is provided with standards graphical user interface (GUI) elements which directly communicate with the model data. The GUI elements are so flexible that their form can be changed by the user. E.g. multivariate tables can be simply rearranged, row or columns lifted up, aggregate rows like sums and counts can be inserted even by the end-user.
- AIMMS is of modular architecture; the model, the actual solver to be used, and the GUI can be selected independenty, yet their interaction is easy to organize. This makes possible developing in case of changing requirements, and coping with the usual needs for further development.
- AIMMS is distributed with standardized communication including standard data base connections, text and Excel form I/O if needed, scripts, APIs, etc.
- AIMMS can be istalled as a normal application with user interface in stand-alone or network version, or as a DLL called by another application so that it serves as an optimization routine only.

Using AIMMS, our standard SCM product is tailored to the particular situations and demands of each enterprise using it. The applications developed in AIMMS are transformed to a so-called end-user form before installing at the user's site. Hierarchical user rights and passwords can be assigned, as usual. AIMMS is also distributing a free-of charge AimmsViewer software. The user of the AimmsViewer can see all the results of the main application in the same form and flexibility as the user of the main application can do; only the essential data cannot be changed. This tool can be of great value if the high management does not use the main application for design but only checks the results without consuming a licence.

Another valuable application is its possible use of collaborative planning in a way that the optimal plan computed by one of the partners is shown to another partner.

3 Further developments and future plans

3.1 Optimal distribution

Optimal distribution includes allocation of goods already produced and stored according to the customers' orders to days and trucks in such a way that each order is satisfied without tardiness and the whole amount is distributed with possible minimum number of trucks and drivers, and a minimum consumption of time and fuel. This normally involves optimal planning of truck routes, considering several customer destinations in a route, constraints on start and end stations as well as time constraints on the drivers, geographic and economic route data, special licences needed to enter some sites, and so on. This is a mixture of optimal distribution, scheduling, and routing, the latter one is related to the travelling salesman problem.

3.2 Collaborative planning

Collaborative Planning, Forecasting, and Replenishment (CPFR) aims at ensuring that there is always enough quantity to meet consumer demand while maintaining optimum levels of stock across supply chain. The essence of collaborative planning is a set of business processes in which business partners agree on mutual business objectives and measures, develop joint operational plans and collaborate to generate and update sales forecasts and replenishment plans.

Forecasting demand (and consequently inventory levels) is complicated due to influence of promotions, changing demand patterns, and competitive pressures. The traditional and simplier answer to inventory problems has been to hold higher inventories. Holding high levels of inventory may offer a way to avoid out-of stocks but avoidance is a very expensive solution.

3.2.1 Focus on improving forecasting

Establishing and influencing demand are critical in every type of business. Every difference between demand and supply can be very costly, resulting in missed sales volume, poor customer service, lost profits and market share, causeless inventory and logistics costs. To maximize business responsiveness and effectiveness, companies usually have accurate forecast, track and analyze customer demand and match product availability with customer orders.

Traditional demand forecasting is based on statistical models that mainly use historical data without incorporating current market driven data, further contributing to demand uncertainty and causing companies to increase inventory, capacity, and logistics costs.

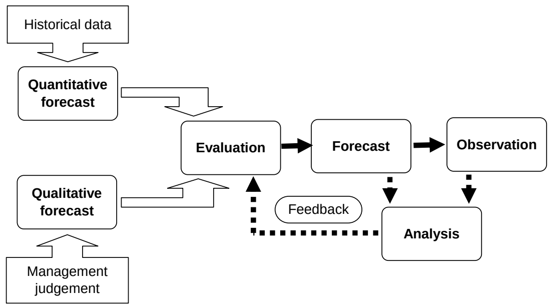

Fig.5. The forecasting system

The basic steps in performing forecasting operation are [4]: (a) Preliminary data analysis (b) Determination of quantitative and/or qualitative forecasts (c) Evaluation and determination of a final forecast (d) Control and feedback.

Collaborative Forecasting is a process in which business partners work interactively to manage forecast requirements by sharing information affecting product management decisions. Forecast exceptions are identified and revisions are exchanged electronically until they are confirmed. In 'Collaborative' Forecasting, business partners work "interactively". The forecast is not simply pushed from a customer to a supplier but forecast information flow has a two-way moving creating a closed loop of communication.

The supplier in a collaborative forecasting system sends a forecast confirmation in reply to a customer's forecast to communicate the supported volume. The overall intent of the process is that the customer will use the forecast response information to create a better forecast in the next iteration of forecast generation. Combining intelligence of both business partners to refine capacity and replenishment plans is the essence of collaborative forecasting.

Since forecasting is very data intensive, the collaborative forecasting process incorporates an exception mechanism that alerts business partners when tolerances or performance thresholds have been exceeded. This "manage by exception" discipline can greatly reduce the effort of supporting a collaborative relationship.

3.2.2 …Interaction of players of the chain

Good communication leads to better planning, and better planning, in turn, leads to good decisions. With global enterprises and supply chains strung out around the world, clear and knowledgeable communications can be problematic but they are paramount to achieving superior business results.

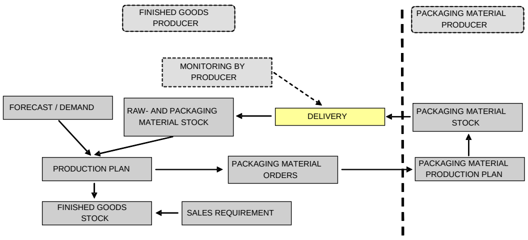

Fig.6. A model from beer industry

Collaborative planning is essential and inevitable, for example, in case of brewery where information share between a beer producer and a manufacturer of cans (see Fig.6) is of key importance. The method called the 'beer game' has been established on the basis of the 'bullwhip effect', which models stocking problems arisen under complicated circumstances. This model has been turned to daily operative and short-term period of time from the classical middle and long-term forecast.

No matter how well forecast is made, the following events can interfere with decision making:

– Delivery inability of the manufacturer of cans due to manufacturing (technical) and inappropriate security stocks
– Service inability of the manufacturer of a finished product due to manufacturing (technical) and inappropriate security stocks
– Inaccurate forecast /demand information or sudden change of market (pro and contra)

The shared information is entirely related to the common field of decision making; hence related to package stock of the manufacturer manufacturing the finished products and its process, in accordance with manufacturing utility and demand for supplementation, sale demand process (demand/forecast) its manufacturing programmes, stock process of the finished products and order of the actual packages. The manufacturer of package continuously updates and introduces manufacturing programs of the package accepted, stocks of package, its stocking process and its own delivery plan for the beer maker on its own manufacturing premises.

Besides regularly harmonizing short-term and long-term programs, in critical cases the partner indicating inbalance introduces reasons of the crisis arisen short-term or in accordance with operative way and its suggestions for solutions (modification of manufacturing programs, replanning of delivery, pre-stocking for consignational stocks, etc). Due to this initiative, the other partner analyzes changes arisen in its own operation and can suggest proposals. Both partners introduce their scenarios for solutions or their iterative series in order the agreement could be made. The more informatics and logistics of the two systems are connected through high added valued tools, the bigger possibilities and chances can be arisen for decision making of collaborative enterprises along their common optimum.

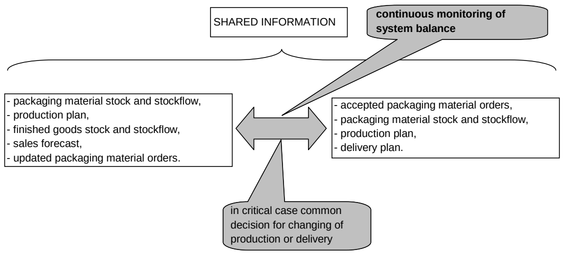

Fig. 7 Information sharing

Among the advantages of the system tighter stock management needs to be emphasized, consequently lower inventory cost level, better cash flow figure, higher level of customer service, bigger flexibility, precise management of collaboration among enterprises independent of the their size (because effective integration of small- and medium enterprises is possible), preliminary analysis of expected difficult situations may also be emphasized. As a consequence, decision making which implies lower risks can be achieved.

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